

Do odd #'s only!

Algebra II

Chapter 2 – Basic Word Problems

Part I – Translating Words to Equations and Inequalities

In this section, you will learn the basics of how to take a group of words and change it to an equation or an inequality. When you have a word problem that you are asked to solve, it is usually best to first read the whole problem to get an idea of what it is that you are being asked to do, and then go back to the beginning and attack it one piece at a time. The table below lists some key words and phrases that you will see in the problems in this section and their meanings.

Word/Phrase	Meaning
x more than y	$x + y$
the sum of x and y	$x + y$
the product of x and y	$x \cdot y$
the quotient of x and y	$\frac{x}{y}$
the difference of x and y	$x - y$
x decreased by y	$x - y$
x less y	$x - y$
x less than y	$y - x$
x fewer than y	$y - x$
is (and most other verbs)	$=$
is more than	$>$
is less than	$<$
is at least	$\geq$
is at most	$\leq$
of (when a fraction, decimal, or percent comes immediately before it)	times

Notice that the phrases "less than" and "fewer than" are weird – they tell you to switch the order from what you would normally write. For instance, you would think that "three fewer than a number" means " $3 - n$," but it actually means " $n - 3$." As another example, you would think that "five less than four times a number" means " $5 - 4n$," but it actually means " $4n - 5$." However, be careful: "x less y" means " $x - y$ " (do not switch the order).

Now, let's look at some problems.

Example 1: Translate to an equation or inequality and solve: Three more than twice a number is at most five times the number.

When we take this problem one piece at a time, we find that it translates to an inequality like this:

Three	more than	twice	a number	is at most	five times	the number
↓	↓	↓	↓	↓	↓	↓
3	+	2 ·	n	≤	5 ·	n

This gives us the inequality $3 + 2n \leq 5n$, which has a solution of $n \geq 1$ (as discussed in Chapter 1).

Example 2: Translate to an equation or inequality and solve: Ten fewer than twice a number is three times the sum of the number and 5.

Recall that the words “fewer than” tell you to switch the order from what you would normally write. We can take the problem one piece at a time and translate it to an equation like this:

Ten	fewer than	twice a number	is	three times	the sum of the number and 5
↙	↓	↘	↓	↓	↓
2n	-	10	=	3 ·	(n + 5)

When we solve this equation, we find that $n = -25$, as shown below.

$$2n - 10 = 3(n + 5)$$

$$2n - 10 = 3n + 15$$

$$-25 = n$$

Use the Distributive Property.

Subtract 15 and 2n from both sides.

Example 3: Your teacher tells you, “Three-fifths of the class made an A on the quiz.” If you know there are 30 people in your class, how many made an A?

We can translate your teacher’s statement to an equation like this:

Three-fifths	of	the class	made	an A on the quiz
↓	↓	↓	↓	↓
$\frac{3}{5}$	·	30	=	a

This tells us that $\frac{3}{5} \cdot 30$, or 18, students made an A.

Example 4: You read in the newspaper that there were 8,700 fans at the game last night rooting for the home team. If it also says that this represented three-fourths of the total number of people there, how many people were not rooting for the home team?

We will focus on part of the second sentence and translate it like this:

this	represented	three-fourths	of	the total number of people there
↓	↓	↓	↓	↓
8,700	=	$\frac{3}{4}$	·	t

Now, we can solve this equation by dividing both sides by $\frac{3}{4}$. This tells us that

$t = 11,600$ (remember your rules for dividing fractions: copy down the first fraction, change the division sign to multiplication, and flip the second fraction). Now, we need to answer the question. Since there were a total of 11,600 fans there, and since 8,700 of them were rooting for the home team, there must have been 2,900 people who were not rooting for the home team (because $11,600 - 8,700 = 2,900$).

Example 5: As a salesperson, you determine that you have talked to 135 people who would not buy from you. Once, while attending a sales conference, a colleague told you, "You should expect that at least 10% of the people you talk to will buy from you." If this person's statement is correct, how many people did buy from you?

This problem is not as easy as the last examples because we cannot take one phrase of information and translate it to an equation or inequality directly. We can, however, rephrase the colleague's statement to say, "You should expect that at most 90% of the people you talk to will not buy from you." Now, the words "at most" say that there will be an inequality in our final answer somewhere, but we will deal with that later. For now, we will put an equal sign in the middle.

Therefore, we translate the colleague's statement like this:

$$\begin{array}{ccccccc} 90\% & \text{of} & \text{the people you talk to} & \text{will} & \text{not buy from you} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.90 & \cdot & p & = & 135 \end{array}$$

When we solve this equation, we find that $p = 150$. This says that you talked to 150 people, and if exactly 10% bought from you, then 15 people bought a product from you (because $150 - 135 = 15$). Now, recall that the colleague said, "at least 10% of the people you talk to will buy from you." If this is true, then we should conclude that at least 15 people bought a product from you (or, in symbolic terms, $b \geq 15$, where b represents the number of people who bought a product from you).

Example 6: You have just poured yourself 10 ounces of a grape drink. If the package says it contains 15% juice, and if you dilute it by adding 1 ounce of water, what percentage is now juice?

To solve this problem, we will first note that 15% of the 10 ounces is juice. This translates to an equation like this:

$$\begin{array}{ccccccc} 15\% & \text{of} & \text{the 10 ounces} & \text{is} & \text{juice} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0.15 & \cdot & 10 & = & j \end{array}$$

This tells us that 1.5 ounces of the mixture is juice. Now, if we add 1 ounce of water, we now have 11 ounces of mixture, but we still have only 1.5 ounces of juice. Since the problem asks us to find the percentage that is now juice, we need to say, "What percent of 11 is 1.5?" and then translate this statement to an equation as shown on the next page.

what percent	of	11	is	1.5
↓	↓	↓	↓	↓
x	•	11	=	1.5

Dividing both sides by 11 tells us that x is approximately 0.136, or about 13.6%.
This tells us that the new mixture is about 13.6% juice.*

Problems – Translate each of the following into an equation or inequality and solve.

1. Three more than twice a number is less than the number. _____
2. Fifteen more than a number is 6 less than twice the number. _____
3. Seventeen less than 5 times a number is at most twice the number. _____
4. Five less than three times the sum of a number and 12 is at least seven times the number. _____
5. Ten less twice a number is more than five times the difference of the number and 15. _____
6. Four more than the quotient of a number and 7 is the same as the product of 3 and 5. _____
7. Four less than twice the product of three and a number is the number less 5.

*Another way of arriving at this last equation is to use the formula $\text{percentage} = \frac{\text{part}}{\text{total}} \cdot 100\%$. For this example, the part is 1.5, and the total is 11. Therefore, we can say $\text{percentage} = \frac{1.5}{11} \cdot 100\% \approx 13.6\%$.

8. A number decreased by 5 is at least 6 more than the number. _____
9. Three more than four times the difference of 5 and a number is 3 more than twice the number. _____
10. Eight times the difference of a number and 2 is at least 4 decreased by the number. _____
11. Robert has at least 7 more than twice the number of baseball cards that his brother Tommy has. If Tommy has 15 baseball cards, how many does Robert have? _____
12. Paula has at most 4 less than twice the number of shoes that her friend Laura has. If Paula has 5 pairs of shoes, how many does Laura have? _____
13. Five less than twice a number is at most two more than twice the sum of the number and 4. _____
14. Two-thirds of the people at the meeting supported the President. If there were a total of 24 people at the meeting, how many supported the President? _____
15. Five percent of the trees in a certain forest are pine trees. If there are at most 50 pine trees in the forest, how many trees are there in all? _____

16. A certain box of cereal says a serving provides 35% of the U.S. Recommended Daily Allowance (USRDA) for Vitamin B₆. If the U.S. Recommended Daily Allowance for Vitamin B₆ is 2 mg, how many milligrams of Vitamin B₆ does a serving of this cereal provide? _____
17. Another cereal box says one serving provides 24 g of carbohydrates, and it also says that this is 8% of the USRDA for carbohydrates. What is the USRDA for carbohydrates? _____
18. The USRDA for riboflavin is 1.7 mg. If a package of vitamins claims to contain 4 mg of riboflavin, about what percentage of the USRDA is this? _____
19. Two of the ships in the fleet were aircraft carriers. If 89.5% of the fleet were **not** aircraft carriers, how many ships were there in all? _____
20. Twenty percent of the people in attendance raised their hands to say they had read the article. If 60 people did not raise their hands, how many people did raise their hands? _____
21. Of the 35 professors in the room, 30 had taught Geometry. About what percentage had **not** taught Geometry? _____
22. Of the people surveyed, only 17% preferred asparagus over carrots. If there were 49 people who preferred carrots, how many people were surveyed? _____

23. You work at Bill's Restaurant. For meats, your customers have a choice of chicken, beef, fish, or none of the above. You have estimated that 37% of your customers prefer beef, 25% prefer fish, 32% prefer chicken, and the rest do not like meat. If your receipts say you served a total of 97 customers yesterday, about how many ordered beef? _____
24. Refer back to #23. If you know that you served some variety of meat to 129 customers today, about how many customers did you serve in all? _____
25. Refer back to #23. You know that you served 253 people this past weekend, and 75 of them ordered beef. How close was your earlier estimate that 37% of your customers prefer beef? _____
26. You have just poured yourself a 6-ounce glass of orange soda. If the carton says it contains 10% juice, and if you add 2 ounces of water to your glass, what percent is now juice? _____
27. Refer back to #26. If, instead of adding 2 ounces of water, you decide to add 2 ounces of pure orange juice, what percent is now juice? _____
28. You are mopping a floor. The carton says you are to dilute the cleaner by using "1 cup to 1 gallon of water." In other words, if you use 1 cup of the cleaner, you are to add 1 gallon of water to it; if you use 2 cups of the cleaner, you are to add 2 gallons of water to it. For this mixture, what is the ratio of cleaner to total mixture? (Hint: There are 16 cups in one gallon.) _____ About what percentage of the mixture is cleaner? _____

29. A carton of bleach says you are to dilute it by adding 1 gallon of water for every $\frac{1}{2}$ cup of bleach. About what percentage of this mixture is bleach? (There are 16 cups in 1 gallon.) _____
30. You work in a pet shop where you had 20 puppies for sale yesterday. If 10% of them were bulldogs, and if one bulldog and one German Shepherd were sold, about what percentage is now bulldogs? _____
31. Three-fourths of the available paper was used in preparing for the presentation. If there were 72 sheets that were not used in preparing for the presentation, how many sheets were used? _____
32. The Bridgeport Wolverines have won 20% of the 10 games they have played so far this season. If they have 16 games left to play, how many more do they have to win to have a winning percentage of 50% for the season? _____
33. Refer back to #32. If the Wolverines go on a sudden winning streak after their first 10 games, how many games do they need to win in a row to raise their overall winning percentage to 60%? _____
34. You are watching a football game where you see that the quarterback has attempted 34 passes and completed 20 of them. About what percentage of his passes has he completed? _____

Part II – More on Translating Words to Equations and Inequalities

The first four examples in this section are called consecutive integer problems. To work these problems, you will need to know that consecutive integers are integers (that is, no fractions or decimals) that are next to each other. In the same way, consecutive even integers are *even* integers that are next to each other, and consecutive odd integers are *odd* integers that are next to each other. Below are some examples that illustrate these.

<u>consecutive integers</u>	<u>consecutive even integers</u>	<u>consecutive odd integers</u>
4, 5, 6	4, 6, 8, 10, 12	1, 3, 5, 7
17, 18, 19, 20	32, 34, 36	151, 153, 155
-1, 0, 1	-2, 0, 2	-1, 1, 3

If you examine the patterns here, you will find that consecutive integers can be represented by $x, x + 1, x + 2, x + 3, \dots$ (where x represents the first integer, $x + 1$ represents the second integer, and so on). Consecutive even and consecutive odd integers can be represented by $x, x + 2, x + 4, \dots$ (where x represents the first integer, $x + 2$ represents the second, and so on).

Example 1: Find three consecutive integers such that five more than the sum of the first and third is three times the second.

We will represent the first integer by x , the second by $x + 1$, and the third by $x + 2$. Now, we can translate this problem to an equation like this:

$$\begin{array}{ccccccc}
 \text{five} & \text{more than} & \text{the sum of the first and third} & \text{is} & \text{three times} & \text{the second} \\
 \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
 5 & + & (x + x + 2) & = & 3 \cdot & (x + 1)
 \end{array}$$

When you solve this equation, you find that $x = 4$. However, the question said to find **three consecutive integers**. Since we know that we represented the first integer by x , the second integer by $x + 1$, and the third integer by $x + 2$, we can conclude that the three integers are 4, 5, and 6.

Example 2: Find two consecutive odd integers such that the first is 17 less than four times the second.

For this example, we will represent the first integer by x , and we will represent the second integer by $x + 2$ (because we want consecutive **odd** integers). This tells us that this problem can be translated to an equation like this:

$$\begin{array}{ccccccc}
 \text{the first} & \text{is} & 17 & \text{less than} & \text{four times the second} \\
 \downarrow & \downarrow & & \downarrow & \downarrow \\
 x & = & 4(x + 2) & - & 17
 \end{array}$$

When we solve this equation, we find that $x = 3$. Since the first integer was represented by x , and the second integer was represented by $x + 2$, we can conclude that the integers are 3 and 5.

Example 3: Find four consecutive even integers such that 26 more than the sum of the first two is two-thirds of the fourth.

Since we want consecutive even integers, we will represent the first by x , the second by $x + 2$, the third by $x + 4$, and the fourth by $x + 6$. This tells us that we can translate this problem to an equation like this:

$$\begin{array}{ccccccc} 26 & \text{more than} & \text{the sum of the first two} & \text{is} & \text{two-thirds} & \text{of} & \text{the fourth} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 26 & + & (x + x + 2) & = & \frac{2}{3} & \cdot & (x + 6) \end{array}$$

When we solve this equation, we find that $x = -18$, as shown below.

$$26 + (x + x + 2) = \frac{2}{3}(x + 6)$$

$$26 + (2x + 2) = \frac{2}{3}x + 4$$

$$28 + 2x = \frac{2}{3}x + 4$$

$$\frac{4}{3}x = -24$$

Subtract $\frac{2}{3}x$ and 28 from both sides.

$$x = -18$$

Remember that $-24 \div \frac{4}{3} = -24 \cdot \frac{3}{4} = -18$.

Now, when we recall that the integers were represented by x , $x + 2$, $x + 4$, and $x + 6$, we can conclude that the integers are -18 , -16 , -14 , and -12 .

Example 4: Find the largest pair of consecutive odd integers such that the first is larger than 11 more than three times the second.

To work this problem, we will begin by translating the second half of the question to an inequality:

$$\begin{array}{ccccccc} \text{the first} & \text{is larger than} & 11 & \text{more than} & \text{three times} & \text{the second} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ x & > & 11 & + & 3 \cdot & (x + 2) \end{array}$$

When we solve this inequality, we get $x < -8.5$. Now, since we want the **largest** pair of consecutive **odd integers**, we can conclude that the first integer must be -9 , and so the two integers must be -9 and -7 .

The next set of examples require you to recall the formulas from Geometry listed below.

perimeter of a rectangle = $2L + 2W$, where L = length and W = width

circumference of a circle = $2\pi r$, where r = radius and π is approximately 3.14

area of a rectangle = $L \cdot W$, where L = length and W = width

area of a circle = πr^2 , where r = radius and π is approximately 3.14

area of a triangle = $\frac{1}{2}bh$, where b (the base) and h (the height) form 90° angles

volume of a prism = Bh , where B = the area of a base and h = the height of the prism

Example 5: The perimeter of a rectangle is 53 inches. If the width is 8 inches, what is the area of the rectangle?

We will begin by translating the first sentence to an equation. To do this, you need to remember that the formula for the perimeter of a rectangle is $P = 2L + 2W$ (where L = length and W = width).

The perimeter of a rectangle is 53 inches

$$2L + 2W = 53 \text{ inches}$$

Now, since we know the width is 8 inches, we can substitute this in for W and solve the resulting equation:

$$2L + 2(8 \text{ inches}) = 53 \text{ inches}$$

$$2L + 16 \text{ inches} = 53 \text{ inches}$$

$$2L = 37 \text{ inches}$$

Subtract 16 inches from both sides.

$$L = 18.5 \text{ inches}$$

Divide both sides by 2.

This tells us that the length is 18.5 inches, and so next we must use the formula $A = L \cdot W$ to find the area of the rectangle:

$$A = L \cdot W$$

$$A = (8 \text{ inches})(18.5 \text{ inches})$$

$$A = 148 \text{ in}^2$$

To multiply 8 inches by 18.5 inches, we multiply 8 times 18.5 and then inches times inches.

Example 6: The area of a circle is $16\pi \text{ m}^2$. What is its circumference?

We translate the first sentence to an equation like this: $16\pi \text{ m}^2 = \pi r^2$. (Recall that the formula for the area of a circle is $A = \pi r^2$.) Now, we can solve this equation as follows.

$$16\pi \text{ m}^2 = \pi r^2$$

$$16 \text{ m}^2 = r^2$$

Divide both sides by π .

$$4 \text{ m} = r$$

Take the square root of both sides.

Next, since our question asked us to find the circumference, we use the formula $C = 2\pi r$:

$$C = 2\pi r$$

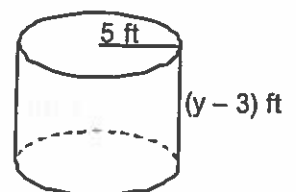
$$C = 2\pi(4 \text{ m})$$

$$C = 8\pi \text{ m}$$

To multiply 2π by 4 m, we multiply the numbers and then keep the meters (because there are no units to multiply the meters by).

Example 7: The volume of the cylinder at the right is $75\pi \text{ ft}^3$. Find the value of y .

To solve this problem, we will use the formula $V = Bh$ (because a cylinder is a prism with a circular base), where B = the area of the base and h = the height of the prism.



In this case, $B = \pi(5 \text{ ft})^2 = 25\pi \text{ ft}^2$ and $h = (y - 3) \text{ ft}$. Since the volume of the cylinder is $75\pi \text{ ft}^3$, we can conclude that $75\pi \text{ ft}^3 = (25\pi \text{ ft}^2)(y - 3) \text{ ft}$. This equation can be solved like this:

$$75\pi \text{ ft}^3 = (25\pi \text{ ft}^2)(y - 3) \text{ ft}$$

$$3 \text{ ft} = (y - 3) \text{ ft} \quad \text{Divide both sides by } 25\pi \text{ ft}^2.$$

$$6 = y \quad \text{Add 3 ft to both sides.}$$

Problems:

35. Find three consecutive integers such that the first is 15 more than the sum of the second and third. _____, _____, _____

36. Find three consecutive odd integers such that the second is 3 less than twice the sum of the first and third. _____, _____, _____

37. Find two consecutive integers such that the second is 12 less than twice the first. _____, _____

38. Find three consecutive even integers such that 7 fewer than three times the second is the sum of the first and third decreased by 5. _____, _____, _____

39. Find four consecutive even integers such that twice the sum of the last two is 46 decreased by the sum of the first two. _____, _____, _____, _____

40. Find two consecutive integers such that the difference of the first and second is 17 more than twice the second. _____, _____

41. Find three consecutive odd integers such that 6 less than 7 times the first is 23 fewer than twice the difference of the third and fourth. _____, _____, _____
42. Find four consecutive even integers such that 7 less than four times the sum of the first three is 125 more than twice the fourth. _____, _____, _____, _____
43. Find two consecutive odd integers such that the first is 51 more than the product of 7 and their sum. _____, _____
44. Find four consecutive integers such that 23 less than the sum of the first two is twice the product of 5 and the sum of the last two. _____, _____, _____, _____
45. Find the largest set of four consecutive even integers such that 4 more than twice the sum of the first two is less than 5 decreased by the sum of the last two. _____, _____, _____, _____
46. Find the largest pair of consecutive integers such that 15 more than the second is at least twice the product of 5 and their sum. _____, _____
47. Find the smallest set of three consecutive odd integers such that 7 less the difference of the first and third is at most 4 more than the second. _____, _____, _____

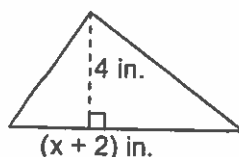
48. Find the smallest set of three consecutive even integers such that -18 less three times the sum of the first two is less than twice the third. _____, _____, _____

49. Find the smallest pair of consecutive integers such that the first is more than 22 decreased by the product of 2 and their sum. _____, _____

50. Find the largest set of four consecutive odd integers such that 12 more than twice the sum of the first and fourth is less than the sum of the second and third. _____, _____, _____, _____

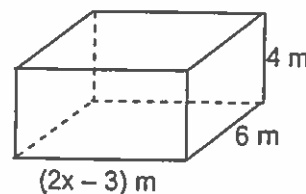
51. The perimeter of a rectangle is 58 centimeters. If the length is 14 centimeters, what is the width? _____

52. The triangle below has an area of 32 in^2 . What is the value of x ? _____

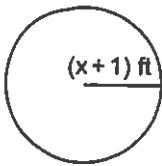


53. If the circumference of a circle is 8π yards, what is its area? _____

54. The volume of the prism at the right is 120 m^3 . What is the value of x ? _____

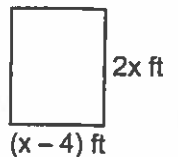


55. The circumference of the circle at the right is 14π feet. What is the length of the diameter of the circle? (Remember that the diameter of a circle is twice the radius.) _____



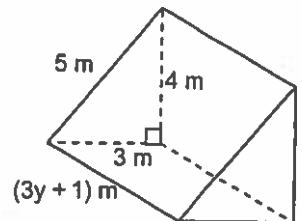
56. The area of a rectangle is 24 ft^2 . If the length is 6 feet, what is its perimeter?

57. If the perimeter of the rectangle at the right is 43 feet, what are the dimensions of the rectangle? _____

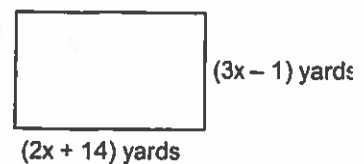


58. The area of a circle is $36\pi \text{ m}^2$. What is the circumference? _____

59. The volume of the prism at the right is 42 m^3 . What is the value of y ? (Hint: Since the bases of a prism must form right angles with the sides of the prism, the bases of this prism are the triangles.) _____



60. If the perimeter of the rectangle at the right is 41 yards, what are the dimensions of the rectangle?



61. If the area of a circle is $25\pi \text{ m}^2$, what is its diameter? _____

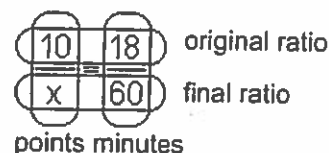
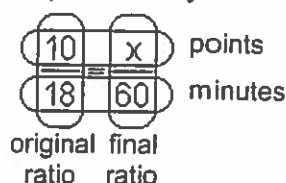
Part III – Ratio Word Problems

In this section, we will look at several types of problems involving ratios. Recall that a ratio is a comparison of two or more quantities. A ratio comparing two quantities can be written in any of the following forms: $\frac{p}{q}$, p to q , or $p : q$. A ratio comparing three or more quantities must be written either as p to q to r to ... or as $p : q : r : \dots$.

Example 1: You are watching a football game on television. You note that, during the first 18 minutes of the game, the two teams have scored a total of 10 points. About how many total points should you expect to see at the end of the game if the game is 60 minutes long?

To solve this problem and the ones in Examples 2 and 3, we will set up a proportion. A proportion is an equation with two fractions (and only two fractions), with an equal sign in the middle. When you write a proportion, you must make sure that the top numbers go together, the bottom numbers go together, the numbers on the left-hand side go together, and the numbers on the right-hand side go together. For this example, this concept gives rise to the proportions $\frac{10}{18} = \frac{x}{60}$ and $\frac{10}{x} = \frac{18}{60}$.

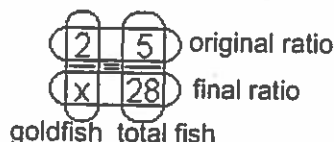
(Either of these proportions will give you the correct answer, and there are also several other ways you can correctly set the proportion up.) You can see how the relationships work by looking at the pictures below.



Now, to finish this problem, we can use the Cross Multiplication Principle discussed in Chapter 1. This gives us $18x = 600$, which tells us $x \approx 33.333$. However, teams cannot score fractions of points, and so, by rounding this number, we can conclude that there will probably be a total of about 33 points scored.

Example 2: As the owner of a pet store, you discover that the ratio of people who prefer goldfish to exotic fish is 2 : 3. If you have space to put out 28 fish, how many of each type should you put out?

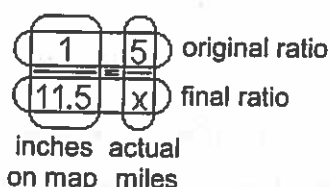
Once again, we will set up a proportion. To solve this problem, you need to realize that the ratio of goldfish to total fish needs to be 2 : 5, and the ratio of exotic fish to total fish needs to be 3 : 5. To find out how many goldfish to set out, you can use the proportion $\frac{2}{x} = \frac{5}{28}$ (although, once again, there are several other ways of setting up the proportion that will give you the correct answer). You can see the various relationships work by looking at the picture below.



Now, we can use the Cross Multiplication Principle discussed in Chapter 1 to say that $56 = 5x$, which tells us that $x = 11.2$. However, since you cannot put out fractions of goldfish, you should put out 11 goldfish and 17 exotic fish (because $28 - 11 = 17$).

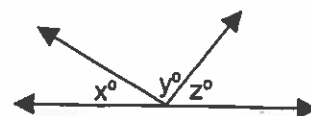
Example 3: The scale on a map says that 1 inch represents 5 miles. If it is 11.5 inches on the map from Oxford to Middletown, how many miles is it from Oxford to Middletown in real life?

Once again, we will set up a proportion. One way we can do this is to write $\frac{1}{11.5} = \frac{5}{x}$. The picture below shows how the relationships work.



This tells us that $57.5 = x$, and so it must be 57.5 miles from Oxford to Middletown in real life.

Example 4: In the picture at the right, the ratio of x to y to z is $2 : 5 : 3$. Find the measures of each of the angles.



To work this problem, the first thing you must realize is that all the angles will add together to give you 180° , since, when you put them all together, they make a straight line. The second thing you must realize is that the ratio $2 : 5 : 3$ is a lot like a reduced fraction: all of the original angle measures have been divided by the same number. We will call that number n . If we "undivide" (i.e., multiply) each of these angle measures by n , then we find that the original angle measures must have been $2n^\circ$, $5n^\circ$, and $3n^\circ$. Now, by using the fact that they must all add to give you 180° , we can say that $2n + 5n + 3n = 180$, which tells us that $n = 18$. Since the original angle measures were represented by $2n^\circ$, $5n^\circ$, and $3n^\circ$, then they must have been 36° , 90° , and 54° . (Incidentally, you could have used this same line of thinking to solve the problem in Example 2. You could have said $2n + 3n = 28$ and solved the problem from there.)

Problems:

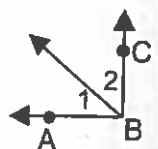
62. A lottery ticket claims that your odds of winning a prize are 1 in 25. (In other words, the ratio of times you win to times you play will probably be about $1 : 25$.) On the average, how many times would you have to play in order to win 6 times?

63. You read in a magazine that you are five times more likely to see a non-poisonous snake than you are to see a poisonous one. What should you expect to be the ratio of times you would expect to see a non-poisonous snake to the total number of snakes? _____
64. Refer back to #63. You estimate that you have seen a total of 32 snakes in your lifetime. About how many times have you seen a poisonous one? _____
65. On a map, it is 13.5 inches from Georgetown to Kelleytown. If the scale says that 2 inches = 5 miles, how many miles is it in real life? _____
66. Your car's manufacturer claims that your car can get 25 miles per gallon in the city (with traffic, stop signs, and traffic lights) and 35 miles per gallon on the highway. How many gallons of gas would you expect to need to drive 400 highway miles? _____
67. Refer back to #66. How many city miles would you expect to be able to drive on 15 gallons of gas? _____
68. When Mark, Andrew, and Gary compared their test scores, they found that the ratio of their scores was 6 : 8 : 9. If the sum of their scores was 253, what were the scores? _____
69. The packaging on model cars often has a scale ratio on it. If the packaging of a model car says it has a ratio of 3 : 32 (where 3 represents the model and 32 represents real life), and if the length of the model car is 17 inches, about how many feet long is the car in real life? _____

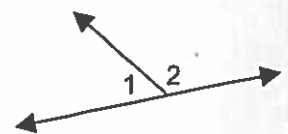
70. The packaging on model airplanes also often has a scale ratio on it. If the real airplane has a wingspan of 70 feet, and if the scale ratio is $\frac{1}{11}$, what should the wingspan of the model be in inches? _____

71. You are the manager of a grocery store. After conducting a survey, you find that 3 out of 5 dog owners prefer Brand A dog food to Brand B dog food. If you have space to put out 20 bags of dog food, how many of each brand should you put out?
Brand A: _____ Brand B: _____

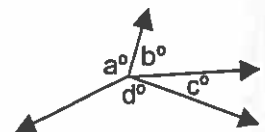
72. In the picture at the right, $\angle ABC$ is a right angle. If the ratio of the measures of $\angle 1$ to $\angle 2$ is $8 : 7$, what is the measure of $\angle 2$? _____



73. In the picture at the right, the ratio of the measures of $\angle 1$ to $\angle 2$ is $4 : 11$. What is the measure of $\angle 1$? _____



74. In the picture at the right, the ratio of a to b to c to d is $5 : 3 : 2 : 8$. What is the value of a ? _____



75. You see that a 3-oz. can of olives is selling for \$0.95. Assuming the same ounces to price ratio, how much would you expect to pay for 5 ounces? _____

76. You see a 16-ounce box of detergent on sale for \$4.19. Assuming the same ounces to price ratio, how much would you expect to pay for 20 ounces? _____

83. For every 2 pitches that the player hit, there were 3 pitches that he could not hit. If the player attempted to hit a total of 60 pitches, how many did he hit? _____
84. Your favorite basketball team has played 4 games so far this season. At the beginning of the fifth game, an announcer says, "So far this season, Joe Brown has attempted 21 free throws and made 17 of them." If the average basketball player plays in 65 games per season, how many free throws would you expect this player to attempt? _____ How many would you expect him to make? _____
85. Refer back to #84. You also see that the score is 16 to 15 after 14 minutes. What would you expect the score to be at the end of the game (after 48 minutes)?

86. You know that it is 62 kilometers from Thomasville to Kingston. As the manufacturer of an atlas, you want to know how far apart the two towns should appear on your map. If you want 2 centimeters to represent 25 kilometers, how many centimeters apart should the towns appear on your atlas? _____
87. Suppose that you stock the shelves at a grocery store. The manager has told you that the ratio of apples to bananas should be 3 : 2. If you have room for a total of 30 of these fruits, how many apples do you need to put out? _____
88. In a factory, the ratio of working machines to non-working machines was 7 : 2. If there were a total of 126 machines in the factory, how many of them were working?

77. You are cooking Cajun-marinated fish. The recipe says that, to cook 4 fish fillets, you should mix the following ingredients together: 1 cup water, $\frac{1}{4}$ cup cooking oil, and 2 tablespoons of Cajun seasoning. How much of each ingredient should you use to cook 10 fish fillets? water: _____; cooking oil: _____; Cajun seasoning: _____
78. You have decided to bake chocolate chip cookies for an upcoming party you will be giving. The recipe says that, to make 18 cookies, you need to mix (among other things) 2 eggs and $\frac{1}{2}$ cup butter. How many eggs and how many cups of butter do you need to make 45 cookies? eggs: _____; butter: _____
79. On a map, it is 2.25 inches from Butler to Fayetteville. If the scale says that 1 inch on the map represents 20 miles in real life, how far is it in real life? _____
80. As you are watching a football game on television, you notice that, after 35 minutes, the home team has 118 rushing yards and 85 passing yards. If a game is 60 minutes long, how many rushing yards would you expect the team to have at the end of the game? _____
81. In another football game, the ratio of a certain quarterback's completed passes to attempted passes is 2 : 3. If the quarterback has **not** completed 12 of his passes, how many has he completed? _____
82. As the manager of an Italian restaurant, you have noticed that, for every 2 cheese pizzas you sell, you sell 5 pepperoni pizzas. If you know that you sold a total of 60 pizzas yesterday, about how many of them were cheese? _____

Part IV – Averages in Word Problems

In this section, we will talk about different types of word problems involving averages. Before we discuss any examples, however, we will review how to find an average. To find an average, you add up all the numbers and then divide by the numbers of things you added. In short, we use the following formula: $\text{average} = \frac{\text{sum}}{\text{number of things}}$. For example, suppose you played golf four times last month. Your scores were 85, 87, 90, and 73. To find the average of these scores, you need to add them up and divide by 4 (because you added 4 scores together). Thus, your average score last month would have been 83.75.

Example 1: The average of three numbers is 17. Find their sum.

To solve this problem, we will use the formula above.

$$\text{average} = \frac{\text{sum}}{\text{number of things}}$$

$$17 = \frac{\text{sum}}{3} \quad \text{We know the average is 17 and the number of things is 3.}$$

$$51 = \text{sum} \quad \text{Multiply both sides by 3.}$$

Hence, the sum of the three numbers must be 51.

Example 2: In a pie-eating contest, the first eight contestants took an average of 2 minutes to eat a pie. The last twelve contestants took an average of 1.75 minutes to eat a pie. What was the overall average for all 20 contestants?

We will begin by looking at what the first two sentences tell us.

first 8 contestants	last 12 contestants	
$2 = \frac{\text{sum}}{8}$	$1.75 = \frac{\text{sum}}{12}$	Recall that $\text{average} = \frac{\text{sum}}{\text{number of things}}$.
$16 = \text{sum}$	$21 = \text{sum}$	

Now, the overall average = $\frac{16 + 21}{20} = 1.85$. Thus, the overall average for all 20 contestants was 1.85 minutes.

Example 3: Eric has the following scores on his tests so far this semester: 85, 78, 91, 95, and 83. What must he score on his next test to raise his average to 88%?

Once again, we will use the formula $\text{average} = \frac{\text{sum}}{\text{number of things}}$, and we will represent the score on the sixth test by x . The problem is solved like this:

$$88 = \frac{85 + 78 + 91 + 95 + 83 + x}{6}$$

$$88 = \frac{432 + x}{6}$$

$$528 = 432 + x$$

$$96 = x$$

Multiply both sides by 6.

Thus, Eric must score a 96 on his next test to raise his average to 88%.

The next example discusses how many teachers calculate the grades for their students. Many teachers use a system called weighted averages. They calculate your average for different categories (such as homework, tests, quizzes, and special projects) separately and individually. Then, they multiply the average by the weight (written as a decimal, not a percent) and add all these numbers together.

Example 4: Nathan's teacher uses the weighted average system. Tests count for 40% of his grade, quizzes count for 30%, homework counts for 25%, and special projects count for 5%. Calculate his grade in the class if his grades are as follows.

Homework: 90, 75, 100, 85, 92, 86

Special Projects: 95

Tests: 85, 72, 91

Quizzes: 99, 95, 76, 64, 71

To work this problem, we start by calculating each of the averages separately. His homework average is 88%, his special projects average is 95%, his test average is about 82.7%, and his quiz average is 81%. Now, we need to multiply these numbers by their weights and add the results together:

$$88(0.25) + 95(0.05) + 82.7(0.4) + 81(0.3) \approx 84\%$$

Therefore, Nathan's current grade in the class is 84%.

Example 5: In Chemistry, the calculation of an element's atomic mass is a weighted average. In nature, chlorine-35 (with a mass of approximately 34.969) occurs 75.78% of the time, and chlorine-37 (with a mass of approximately 36.966) occurs 24.22% of the time. What is the approximate atomic mass of naturally occurring chlorine?

To calculate the atomic mass of naturally occurring chlorine, we multiply the atomic masses by their relative abundances and add the results together:

$$34.969(0.7578) + 36.966(0.2422) \approx 35.453$$

Hence, the atomic mass for naturally occurring chlorine is approximately 35.453.

Problems:

89. The average of n numbers is 74. If their sum is 518, what is the value of n ? _____

90. Find the average of the following set of numbers: 34, 21, 85, 26, and 7. _____

91. The average of 7 numbers is 9, and the average of 9 different numbers is 7. What is the overall average of all 16 numbers? _____

92. At a local fair, the weights of the first 10 pigs averaged 800 pounds. The weights of the next 40 averaged 900 pounds, and the weights of the last 50 averaged 950 pounds. Find the overall average weight of all 100 pigs. _____
93. Jeremy's first 4 grades averaged 82, and his last 6 grades averaged 92. Find the overall average of all 10 grades. _____
94. The average of 6 numbers is 30, and the average of 14 other numbers is 10. Find the overall average of all the numbers. _____
95. The average of Timothy's last five bowling scores is 73.4. What is the sum of his these scores? _____
96. Refer back to #95. What score does Timothy need to get in his next game to have an overall average of at least 85? _____
97. Gail's homework grades are as follows: 95, 90, 0, 91, and 85. What does she need to score on her next homework in order to have a homework average of at least 76? _____
98. Paula's homework grades are as follows: 76, 89, 88, 96, and 87. What does she need to score on her next homework in order to have a homework average of at least 85? _____
99. So far this semester, Al has taken 3 tests, and his average for these tests is 75. What must he average on the next two tests to raise his overall average to 80?

100. The Hamilton Bears have played eight baseball games so far this season. Over the course of these eight games, Peter has gotten a total of ten hits. If the season is twenty games long, how many more hits does he need in order to average three hits per game? _____
101. Refer back to #100. Sam also plays for the Hamilton Bears, and, over the course of the first eight games, he has averaged two hits per game. How many more hits does he need to finish the season with an average of one hit per game? _____
102. Refer back to #100. Kevin also plays for the Hamilton Bears, and, over the course of the first eight games, he has averaged 1.5 hits per game. How many hits does he need to average over the next 12 games in order to have an overall average of 3 hits per game? _____
103. A baseball batting average is computed by dividing the number of hits by the number of times the player has tried to get a hit (called the number of "at-bats"). For example, if a player gets a hit 3 times after 8 at-bats, the player's average is 0.375 (because $3 \div 8 = 0.375$). In his first seven games of the season, Henry has gotten 9 hits, and his average is 0.300. How many at-bats has he had? _____
104. Refer back to #103. If Henry wants to raise his overall average to at least 0.325, what does his average need to be over the course of his next 50 at-bats? _____
105. Refer back to #103. If Henry wants to keep his overall average above 0.250, what does his average need to be over the course of his next 50 at-bats? _____

106. So far this season, Kim has had 40 at-bats, and her average is 0.250. If she goes on a sudden hitting streak where she gets a hit every time she comes up, how many more hits does she need to get her average up to at least 0.375? _____
107. Rafael's batting average so far this season is 0.232, and he has 29 hits. If he goes on a sudden hitting streak where he gets a hit every time he comes up, how many more hits does he need to raise his average to 0.360? _____
108. In the four high school basketball games she has played in during the past week, Nancy has averaged 25 points. Over the course of the next three games, how many points does she need to average if she wants to raise her overall average to 28 points per game? _____
109. Dawn's teacher uses the weighted average system to compute her grade. Tests count for 35%, quizzes count for 30%, homework counts for 20%, and the final exam counts for 15%. Her grades are as follows: Tests: 60, 85, 79, 80; Quizzes: 75, 98, 63, 76, 75, 83; Homework: 80, 90, 76, 87, 76, 91, 95; Final Exam: 73. Calculate her grade. _____
110. Robert's teacher also uses the weighted average system to compute his grade. Tests count for 40%, quizzes count for 25%, homework counts for 20%, special projects count for 5%, and the final exam counts for 10%. His grades are as follows: Tests: 71, 75, 75, 82; Quizzes: 62, 78, 85, 72, 78; Homework: 92, 0, 53, 83, 71, 83, 72; Special Projects: 90; Final Exam: 80. Calculate his grade. _____
111. Paul's teacher counts his final exam as 25% of his grade, and everything else counts for 75% of his grade. If his teacher tells him, "Your average before the exam is 83," what is the lowest grade he can score on the exam and still have an overall average of at least 80? _____

112. Refer back to #111. What is the lowest grade he can score and have an overall average of at least 85? _____
113. Calculate the approximate atomic mass of sulfur if sulfur-32 (with a mass of approximately 31.972) occurs 94.93% of the time, sulfur-33 (with a mass of approximately 32.971) occurs 0.76% of the time, sulfur-34 (with a mass of approximately 33.968) occurs 4.29% of the time, and sulfur-36 (with a mass of approximately 35.967) occurs 0.02% of the time. _____
114. Calculate the approximate atomic mass of magnesium if magnesium-24 (with a mass of approximately 23.985) occurs 78.99% of the time, magnesium-25 (with a mass of approximately 24.986) occurs 10.00% of the time, and magnesium-26 (with a mass of approximately 25.983) occurs 11.01% of the time. _____
115. After playing 9 holes of golf, Thomas, Tonya, and Xavier decided they needed a break. When they sat down and compared their scores, they found that their scores were in the ratio 2 : 3 : 4. If the average of their scores was 54, what were each of their scores? _____
116. When Wendy, Bert, and Jackie compared their bowling scores, they found that the ratio of their scores was 2 : 3 : 5. If the average of their scores was 120, what were their scores? _____
117. During their first five games of the season, the Springfield Cougars scored 1 point, 3 points, 4 points, 2 points, and 6 points. A friend tells you that they are averaging 8 points per game. **Without doing any calculations**, tell why this must be incorrect. _____
-

Part V – More Word Problems

In this section, we will review two more types of word problems: percent increase and decrease problems and problems that use the formula $\text{distance} = (\text{rate})(\text{time})$.

To work percent increase and percent decrease problems, you will need to know the following two formulas.

$$\text{final amount} = \text{original amount} + (\text{percent increase})(\text{original amount})$$

$$\text{final amount} = \text{original amount} - (\text{percent decrease})(\text{original amount})$$

If the final amount represents an increase over the original amount, you will need to use the first formula; if the final amount represents a decrease over the original amount, you will need to use the second formula.

Example 1: As the manager of a car dealership, you want to mark the price of a new car up 10% over the price you pay the factory. If the price you pay the factory is \$18,000, what should the price of the car be?

This example is a percent increase problem (because the final price will be higher than the original price). Therefore, we will use the first formula. The original amount is \$18,000, the percent increase is 10% (although we will write it as 0.10 when we substitute it into the formula), and we are trying to find the final price. This gives us the following:

$$x = 18,000 + (0.10)(18,000)$$

$$x = 19,800$$

This tells us that the final price should be \$19,800.

Example 2: You have a coupon for 15% off an oil change on your car. If the cashier asks you to pay \$16.99 (not including the sales tax), what was the original price of the oil change?

In this case, we will use the percent decrease formula. We know that the final amount is \$16.99, the percent decrease is 15%, and we want to know the original amount. Now, we can substitute the numbers into the formula and solve the resulting equation:

$$16.99 = x - (0.15)(x)$$

$$16.99 = 0.85x$$

$$19.988 \approx x$$

Note that $x - 0.15x = 0.85x$

Divide both sides by 0.85.

This tells us that the original price of the oil change must have been \$19.99, not including the sales tax.